Improved approximating model of hysteresis loop for the linearization of a probe microscope piezoscanner

Rostislav V. Lapshin, email: rlapshin@gmail.com

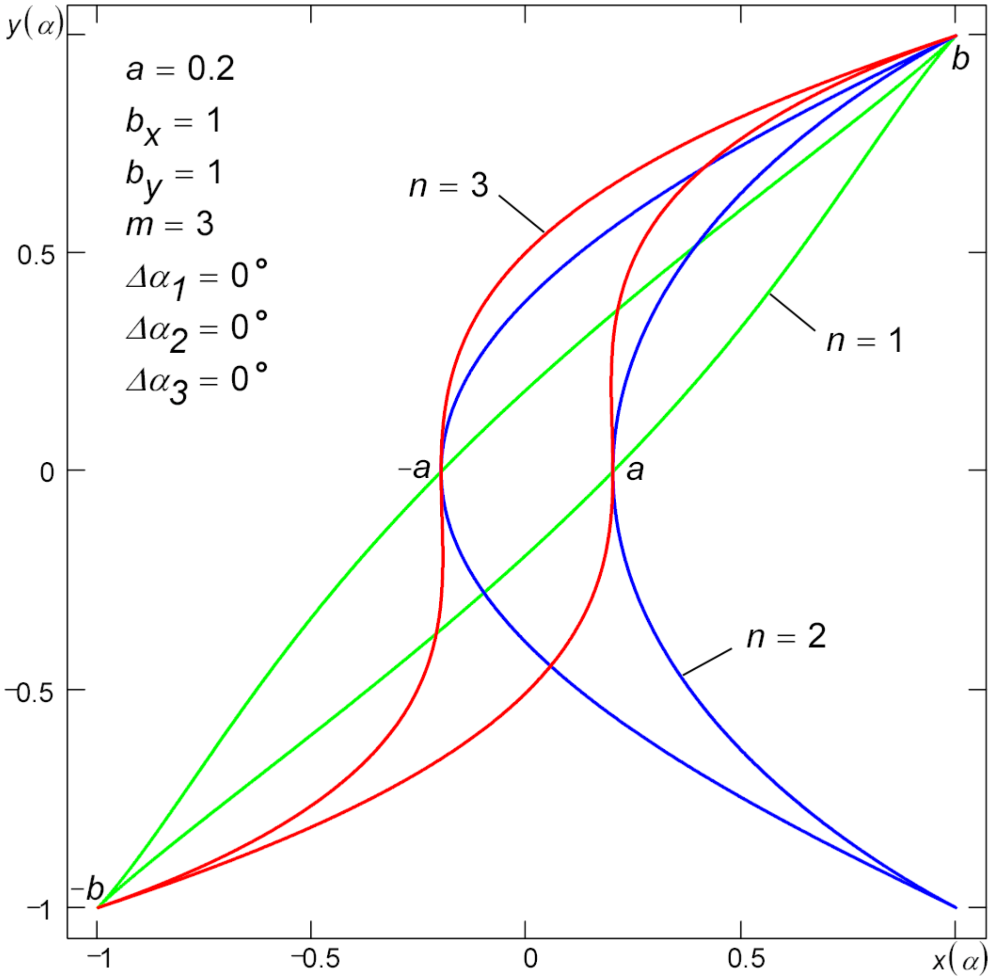
Institute of Physical Problems, Zelenograd, Russia
Moscow Institute of Electronic Technology, Zelenograd, Russia

Chernogolovka – 2015

Abstract

The suggested model covers most of the known types of symmetrical hysteresis loops, and allows for the building of smooth, piecewise-linear, hybrid, mirror-reflected, inverse and double loops. The improvement introduces phase shifts $\Delta\alpha_1$, $\Delta\alpha_2$, $\Delta\alpha_3$ into the existing model. The phase shift $\Delta\alpha_1$ makes it possible to change the loop tilt at the split point. The phase shifts $\Delta\alpha_2$, $\Delta\alpha_3$ allow for continuously changing the loop curvature. The model is simple and intuitive; it permits quickly creating hysteresis loops of a required type and easily defining their parameters. The relative error in approximating a hysteresis loop is about 1%.

Parametric equation of a family of hysteresis loops



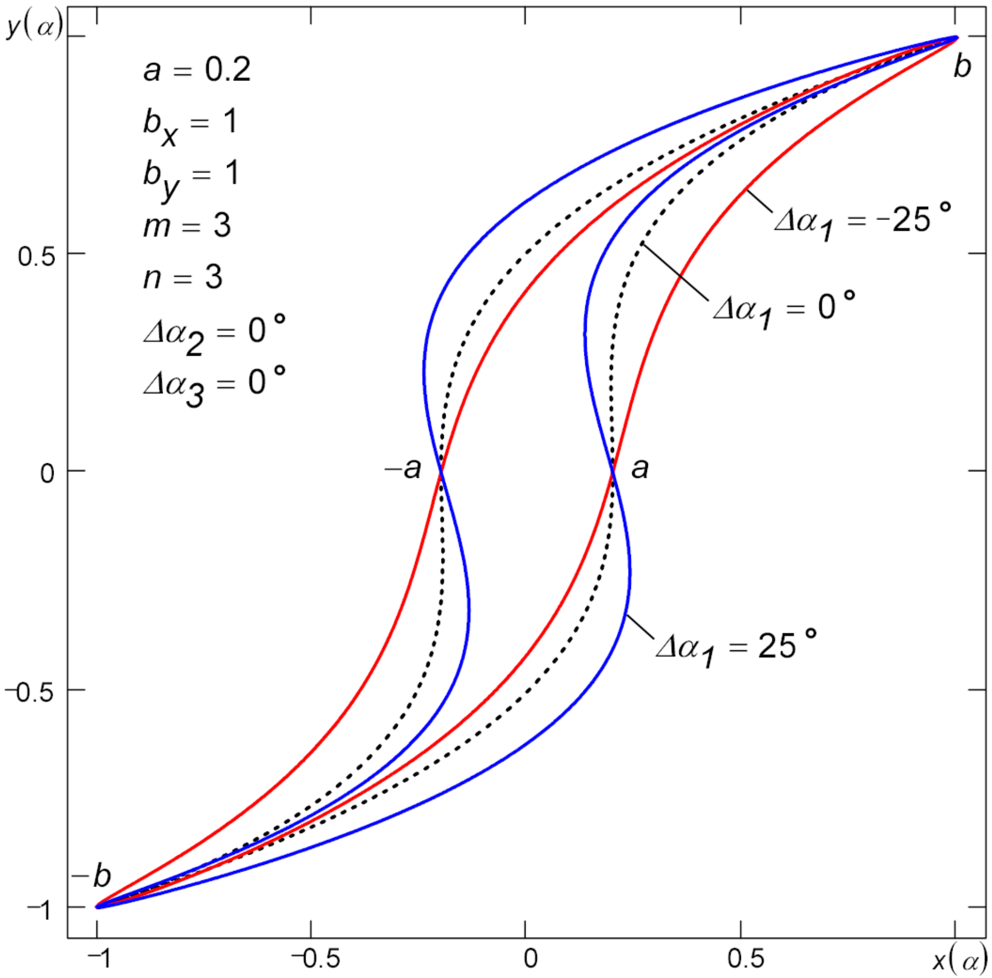
$$x(\alpha) = a\cos^m \alpha + b_x \sin^n \alpha,$$

$$y(\alpha) = b_y \sin\alpha,$$

where α is a parameter (α=0…2π); a is x coordinate of the split point; b_x , b_y are the saturation point coordinates; m is an integer odd number (m=1, 3, 5, …) defining the curvature of the loop; n is an integer defining the type of the hysteresis loop (with n=1, the “Leaf” loop type is formed; with n=2 – the “Crescent”, and with n=3 – the “Classical”)

Introduction of phase shifts $\Delta\alpha_1$, $\Delta\alpha_2$, $\Delta\alpha_3$

$$x(\alpha) = a^c \cos^m(\alpha + \Delta\alpha_1) + b_x^c \sin^n(\alpha + \Delta\alpha_2),$$
$$y(\alpha) = b_y \sin(\alpha + \Delta\alpha_3),$$

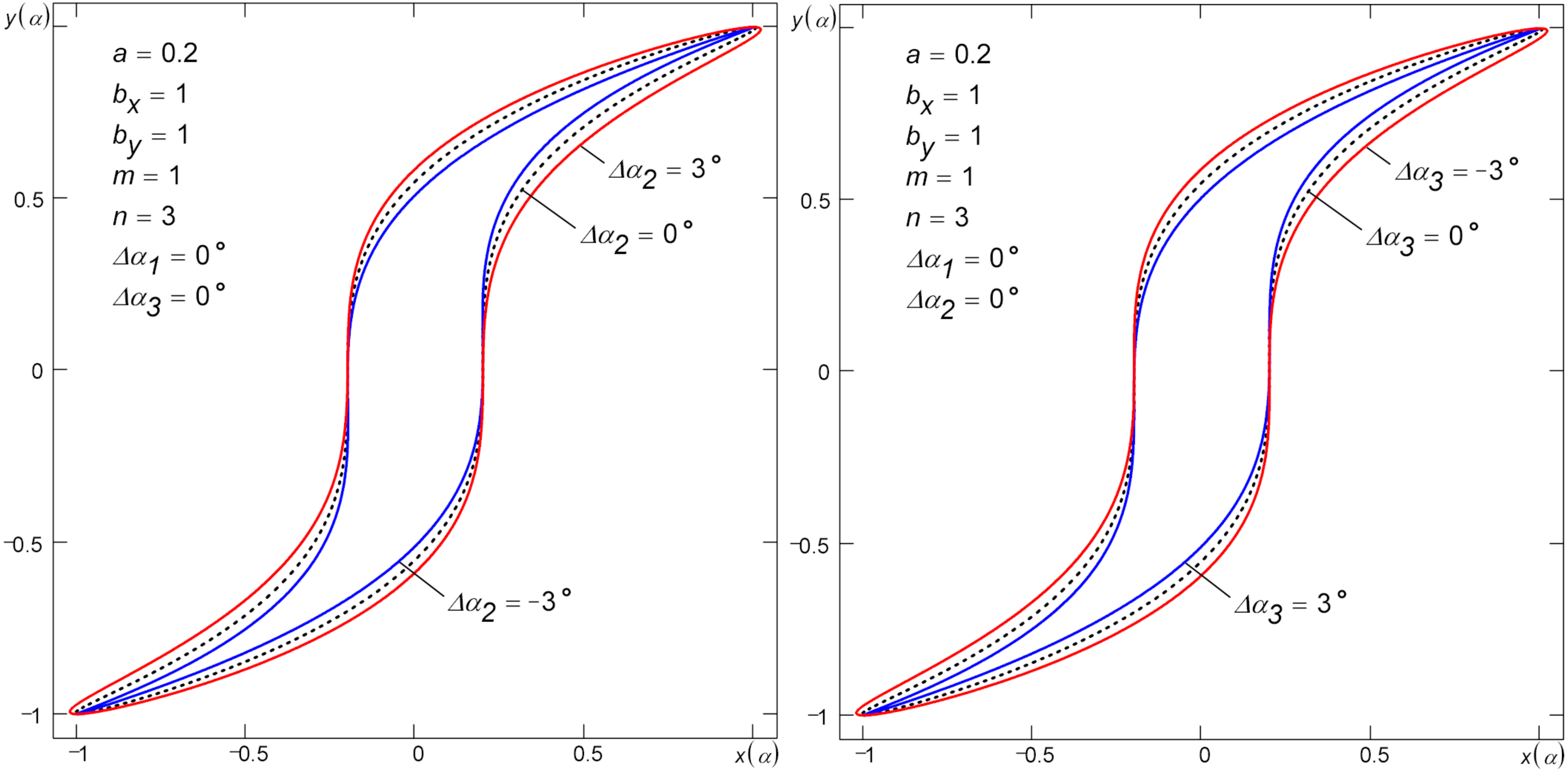


where a^c, b_x^c are corrected parameters of a, b_x

The phase shift $\Delta\alpha_1$ allows tilting a hysteresis loop at the split point a

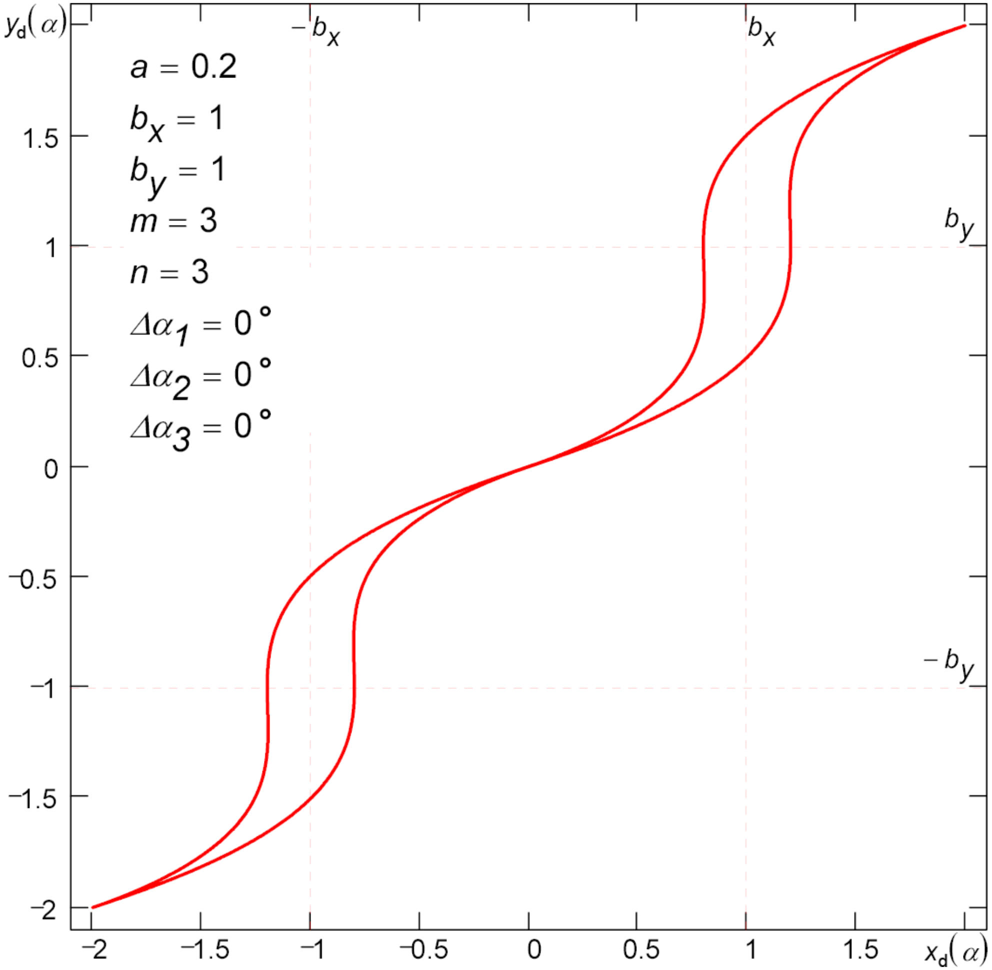
Effect of the phase shifts $\Delta\alpha_2$, $\Delta\alpha_3$

The phase shifts $\Delta\alpha_2$ and $\Delta\alpha_3$ provides a continuous change of the loop curvature



Additional capabilities of the model

Double loops



$$x_d(\alpha) = x\left[2\alpha - (-1)^{\mathrm{rnd}\left(\frac{\alpha}{2\pi}\right)}\frac{\pi}{2}\right] + (-1)^{\mathrm{rnd}\left(\frac{\alpha}{2\pi}\right)} b_x,$$

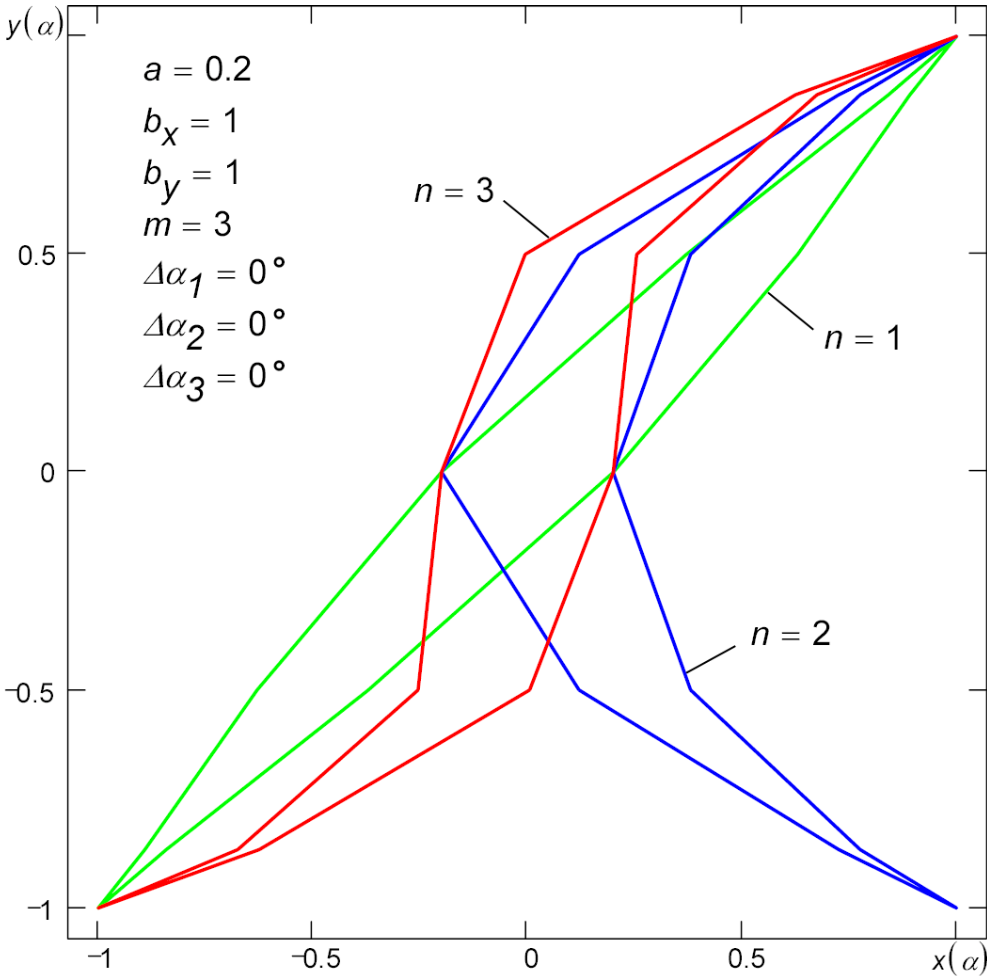
$$y_d(\alpha) = y\left[2\alpha - (-1)^{\mathrm{rnd}\left(\frac{\alpha}{2\pi}\right)}\frac{\pi}{2}\right] + (-1)^{\mathrm{rnd}\left(\frac{\alpha}{2\pi}\right)} b_y,$$

where rnd() is a function rounding to the nearest integer

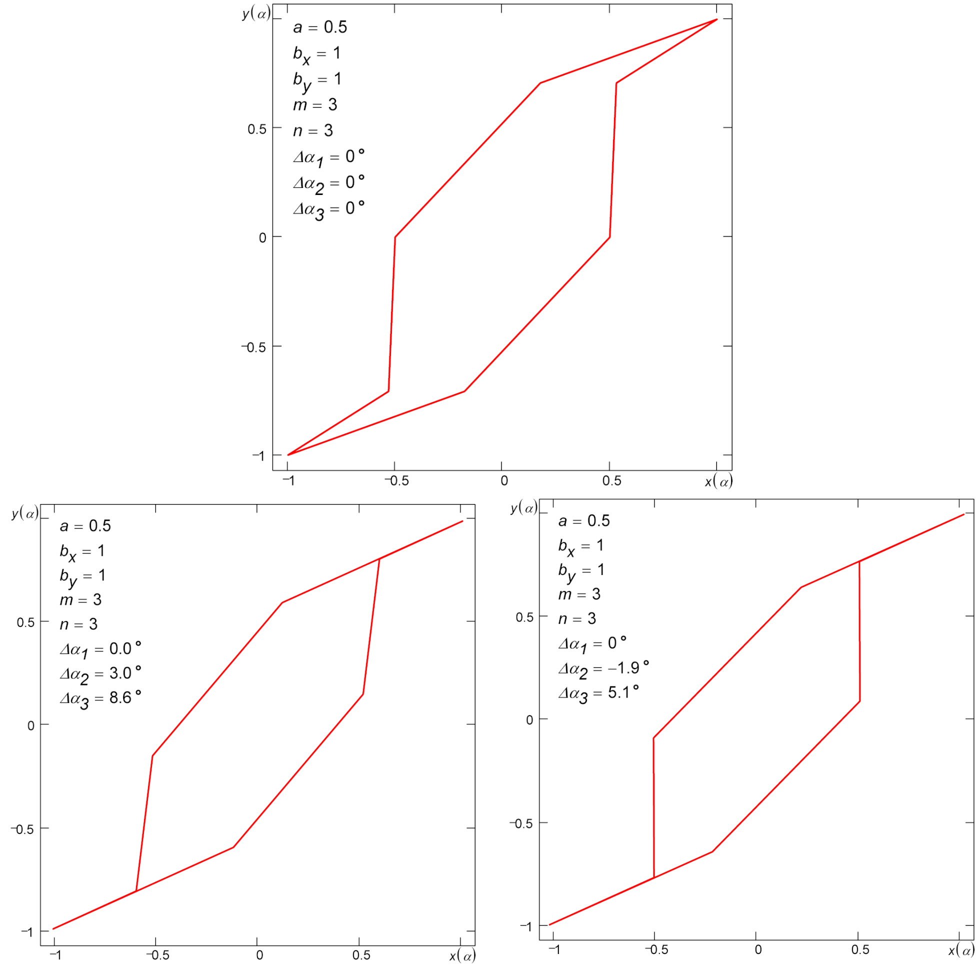
Piecewise-linear loops

The parameter α takes values from 0 to 2π with step $2\pi/k$, where k is an even integer ($k \geq 4$)

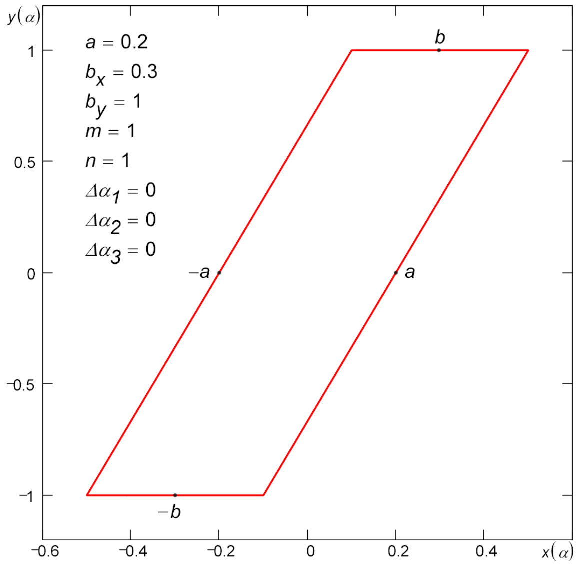
k=12



k=8



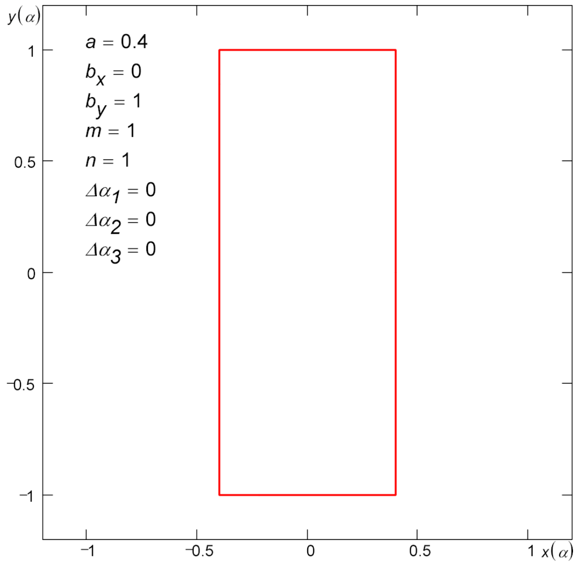
Piecewise-linear loops built by means of trapezoidal pulses

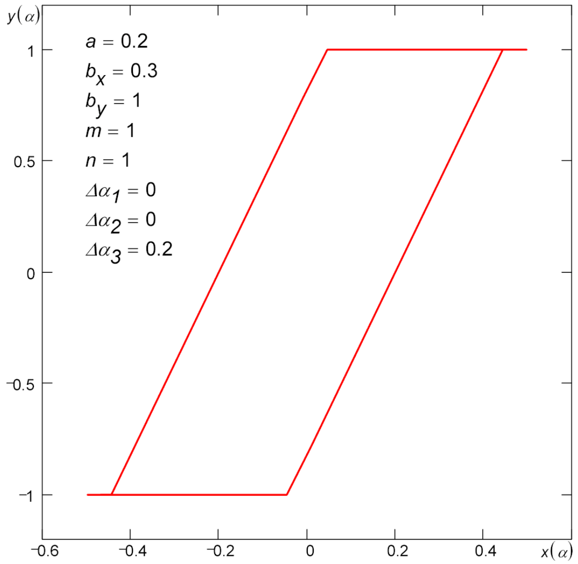


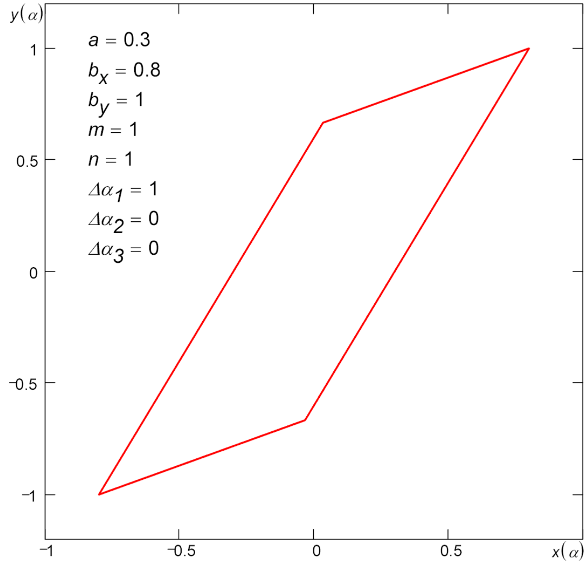
$$x(\alpha) = a^c \operatorname{trp}_c^m(\alpha + \Delta\alpha_1) + b_x^c \operatorname{trp}_s^n(\alpha + \Delta\alpha_2),$$

$$y(\alpha) = b_y \operatorname{trp}_s(\alpha + \Delta\alpha_3),$$

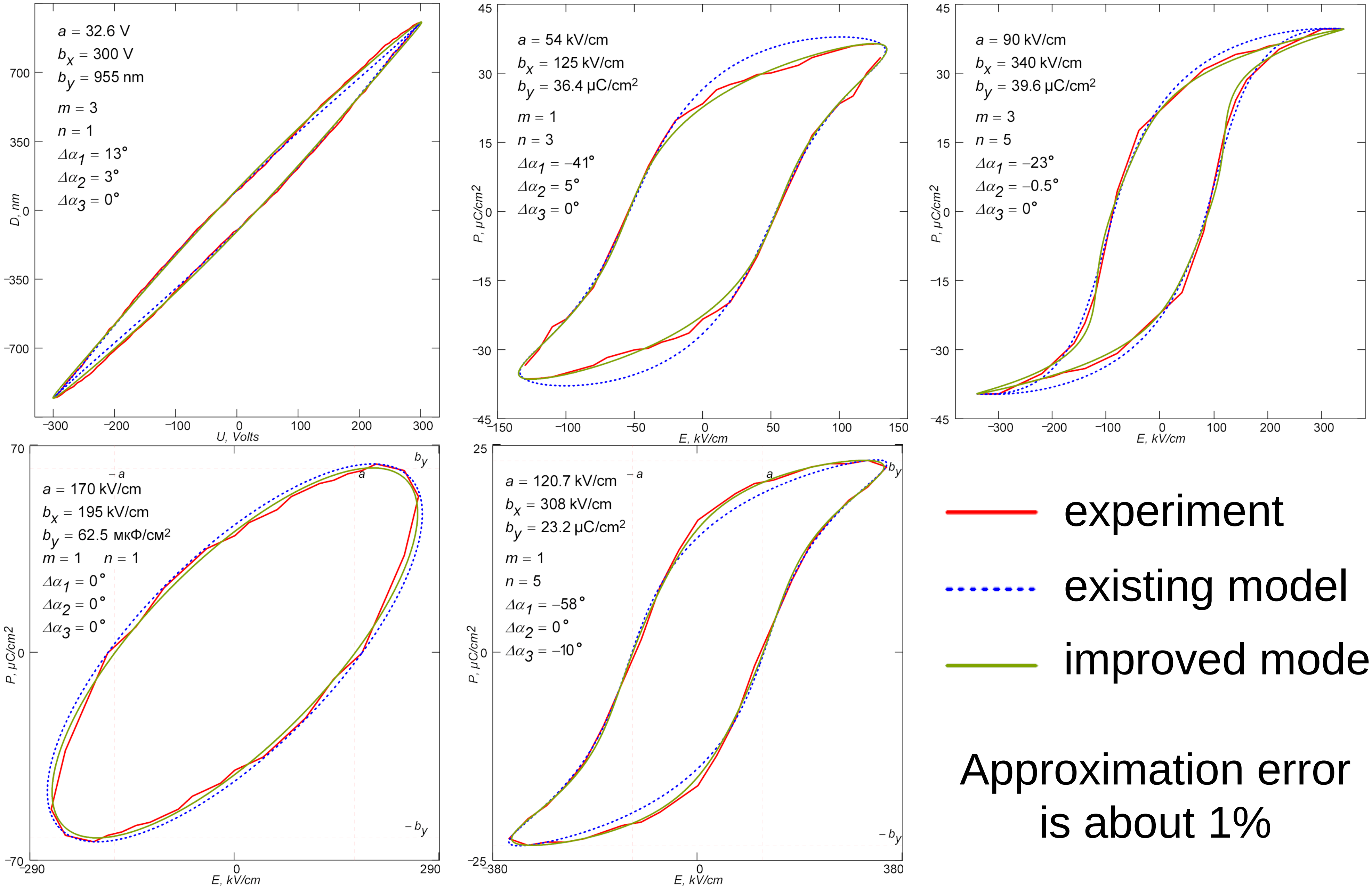
where trp are trapezoidal pulses ($\operatorname{trp}_c(\alpha)=\operatorname{trp}_s(\alpha+T/4)$, T is a period)







Examples of approximation of real loops



experiment

existing model

improved model

Approximation error is about 1%

Areas of application

- Linearization of piezoceramic scanners and manipulators
- Linearization of magnetic and magnetostrictive scanners and manipulators
- Simulation of instruments that include hysteresis elements

References

1. R. V. Lapshin, Analytical model for the approximation of hysteresis loop and its application to the scanning tunneling microscope, *Review of Scientific Instruments*, vol. 66, no. 9, pp. 4718-4730, 1995 (www.niifp.ru/staff/lapshin/en/#articles)
2. Supplementary materials: R. V. Lapshin, Hysteresis loop, Mathcad worksheet, 2015 (www.niifp.ru/staff/lapshin/en/#downloads)
3. S. A. Agafonov, V. A. Matveev, Dynamics of a balanced rotor under the action of an elastic force with a hysteresis characteristic, *Mechanics of Solids*, vol. 47, no. 2, pp. 160-166, 2012

Appendix

Formulae for calculation of the corrected parameters

$$a^c = \frac{a\cos^n(\Delta\alpha_2 - \Delta\alpha_3) - b_x \sin^n(\Delta\alpha_2 - \Delta\alpha_3)}{\sin^m(\Delta\alpha_1 - \Delta\alpha_3)\sin^n(\Delta\alpha_2 - \Delta\alpha_3) + \cos^m(\Delta\alpha_1 - \Delta\alpha_3)\cos^n(\Delta\alpha_2 - \Delta\alpha_3)},$$

$$b_x^c = \frac{a\sin^m(\Delta\alpha_1 - \Delta\alpha_3) + b_x \cos^m(\Delta\alpha_1 - \Delta\alpha_3)}{\sin^m(\Delta\alpha_1 - \Delta\alpha_3)\sin^n(\Delta\alpha_2 - \Delta\alpha_3) + \cos^m(\Delta\alpha_1 - \Delta\alpha_3)\cos^n(\Delta\alpha_2 - \Delta\alpha_3)}.$$